11/12/19

MLS 9 (Continued)

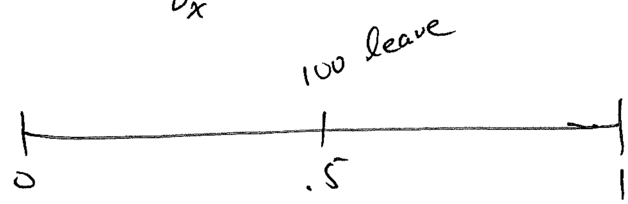
Discrete Decrements Examples

19) Since (1) is discrete (MOY), start with it.

Suppose $l_x = 1000$

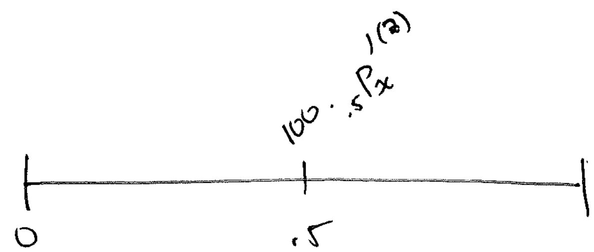
Then $d_x'^{(1)} = 1000 \cdot q_x'^{(1)} = 100$

$d_x'^{(1)}$:



However:

$d_x^{(1)}$:



$$\therefore \frac{d_x^{(1)}}{l_x} = \frac{\cancel{1000}\, l_x \cdot {}_{.5}p_x'^{(2)} \cdot q_x'^{(1)}}{\cancel{l_x}}$$

$q_x'^{(1)} = 0.1$

$q_x'^{(2)} = 0.3$

$$q_x^{(1)} = {}_{.5}p_x'^{(2)} \cdot q_x'^{(1)}$$

$$_{.5}p_x'^{(2)} = 1 - .5 \cdot q_x'^{(2)} = 0.85$$

$$\therefore q_x^{(1)} = (.85)(0.1) = \cancel{0.8}\ 0.085$$

Use totals to get $q_x^{(2)} = 0.285$

19) (Alt. Solution) (1) is MOY

(2) is SUDD

$$q_x'^{(1)} = 0.1$$

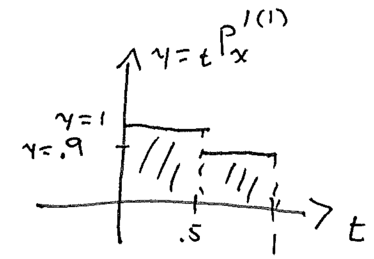
$$q_x'^{(2)} = 0.3$$

Generally $q_x^{(2)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(2)} \cdot dt$

$$= \int_0^1 {}_tp_x'^{(1)} \cdot \underbrace{{}_tp_x'^{(2)} \cdot \mu_{x+t}^{(2)}}_{= \text{constant} = q_x'^{(2)} = 0.3} dt$$

$$\therefore q_x^{(2)} = 0.3 \cdot \int_0^1 {}_tp_x'^{(1)} dt$$

$${}_tq_x'^{(1)} = \begin{cases} 0 & t < 0.5 \\ q_x'^{(1)} = 0.1 & \cancel{t \to 0.5} \ \ 0.5 < t < 1 \end{cases} \Rightarrow {}_tp_x'^{(1)} = \begin{cases} 1 & t < 0.5 \\ 0.9 & 0.5 < t < 1 \end{cases}$$



$$\Rightarrow \int_0^1 {}_tp_x'^{(1)} dt = .5 + \cancel{.9}\,.45 = .95$$

$$\therefore q_x^{(2)} = 0.3\,(0.95) = 0.285$$

Use totals to get $q_x^{(1)} = 0.085$

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20) (1) is SUDD

(2) is Discrete (25% @ t=.3 ; 75% @ t=.7)

$q_x'^{(1)} = 0.2$ $\quad q_x'^{(2)} = 0.4$

Method 1: start w/ (2) since it's discrete

$$q_x^{(2)} = {}_{.3}p_x'^{(1)} \cdot (.25\, q_x'^{(2)}) + {}_{.7}p_x'^{(1)} \cdot (.75\, q_x'^{(2)})$$

$$= (1 - .3(0.2))(.25(.4)) + (1 - .7(.2)) \cdot (.75(.4)) = 0.352$$

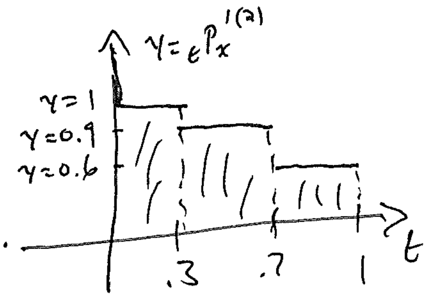
Use totals to get $q_x^{(1)}$.

Method 2: Suppose $l_x = 1000$

$$d_x^{(2)} = 400 \quad (100 \text{ depart @ } t=.3;\ 300 \text{ depart at } t=.7)$$

$$ {}_tq_x'^{(2)} = \begin{cases} 0 & \text{if } t<.3 \\ 0.1 & \text{if } .3<t<.7 \\ 0.4 & \text{if } .7<t<1 \end{cases}$$

$$\therefore\ {}_tp_x'^{(2)} = \begin{cases} 1 & \text{if } t<.3 \\ 0.9 & \text{if } .3<t<.7 \\ 0.6 & \text{if } .7<t<1 \end{cases}$$



$$q_x^{(1)} = \int_0^1 {}_tp_x'^{(2)} \cdot \underbrace{{}_tp_x'^{(1)} \cdot \mu_{x+t}^{(1)}}_{\underline{\text{SUPP}} \text{ constant } = q_x'^{(1)} = 0.2} dt$$

$$= 0.2 \cdot \int_0^1 {}_tp_x'^{(2)}\, dt$$ use areas under the above graph

$$= 0.2\big(0.3 + (.4)(.9) + (.3)(.6)\big) = 0.168$$

MISIO: Multi-State Models
(generalizes previous material)

Example 1: Single-life Single-Decrement (death) Model

State 0: (x) is still alive

State 1: (x) has died

Picture $\quad (0) \longrightarrow (1)$

Notation: μ_x^{01} = force of transition from state 0 to state 1

$\neq 0$ since (x) can transition from (0) to (1)

$\mu_x^{10} = 0$ since (x) cannot ~~from~~ transition from (1) to (0)

Note: There is no μ_x^{ii} (no force from a state to itself)

These forces are used (as before) to calculate probabilities

Notation:

${}_nP_x^{ij}$ = Pr(a person in state i at age x will be in state j at age x+n)

In the model aboove

$${}_nP_x^{00} = {}_nP_x$$

$${}_nP_x^{01} = {}_nq_x$$

$${}_nP_x^{10} = 0$$

$${}_nP_x^{11} = 1$$